

Transient Radiation of a Single-cycle Sinusoidal Pulse from a Thin Dipole

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ABSTRACT: *The behavior of the radiated electric field from a half-wave dipole in free space excited by a single-cycle sinusoidal voltage is studied. The zero-order approximate solution for the current along the antenna in the frequency domain is used, so that the time-dependent current as well as the radiated electric field can be calculated and interpreted easily. Under a suitable matched condition, the time dependent current along each half-section of the dipole is made up of a current pulse traveling from the feed-point and a reflected one from the end-point. However, the radiated electric field consists of five overlapping signals exhibiting that radiation takes place only from the discontinuities of the dipole antenna. Both current and the radiated electric field are extended in time due to the reflection of current from the end-points of the dipole antenna. It appears that the spectral bandwidth of the radiated electric field is narrower than that of the exciting voltage pulse. This phenomenon may be due to the filtering effect of the half-wave dipole.*

I. Introduction

Several radar systems use non-sinusoidal or carrier-free pulses for the detection and identification of targets, since the radiated signals have a broad frequency bandwidth which may be helpful in target detection. Since a successful identification of a target may require the knowledge of the shape of the returned pulses, it becomes important to know beforehand the shape of the incident waveform on the target, remembering the fact that this incident waveform is not the same as the pulse applied to the transmitting antenna. Once the shape of the incident pulse (which is the radiated signal from the transmitting antenna) is known, then the returned or scattered signal from the target can be determined in principle. It appears that a single-cycle sinusoidal signal also has a broadband property. In addition, since the shape of the radiated signals from an antenna excited by a sinusoidal pulse may be easier to recognize, it is desired to study the behavior of a radiated signal from such an exciting source. Although transient radiation from antennas has been studied extensively in the literature (1, 2) using various shapes of the exciting pulses, it appears that the radiation from an electric dipole excited by a single-cycle sinusoidal voltage pulse has not been investigated previously. Since Refs (1, 2) discuss several methods of analysis of transients problems and provide exhaustive related references, there is no need to review different methods and models here.

In general, a transient problem of this type cannot be solved rigorously by any analytical method. However, a fairly accurate result can be obtained numerically (3, 4), which is also time consuming. For this reason we shall use some approximations which will enable us to calculate the time-dependent radiated electric field analytically. This approximation consists of the assumption of a thin antenna, where the current appears to be concentrated along the antenna axis. In addition, only the zero-order solution of the integro-differential equation for the antenna current in the frequency or Fourier-transform domain is retained using these approximations. By allowing a non-zero internal impedance (real) of the pulse generator, a modified zero-order antenna current in the frequency domain is re-calculated. Then the time-dependent antenna current as well as the radiated electric field pulses are determined. When the generator impedance is matched to that of the dipole, it is found that on each half-length of the antenna there are two signal currents, one starts directly from the center of the feed-point and the other arrives after undergoing a single reflection from the end-point of the antenna. Due to this reflection, both the total signal current and the total radiated electric field are extended in time. The electric field consists of five overlapping signals for all angles of observation, exhibiting the fact that radiation takes place only from the discontinuities (feed- and end-points) of the antenna. Since two of these signals have the same phase, only four signals are apparent. Furthermore, the total current at a given point $|z|$ on the antenna shows an even symmetry about the point $f_0 t = 1.5/2$, whereas the total radiated electric field has an odd symmetry about normalized time $f_0 t^* = 1.5/2$, where f_0 is the frequency of the sinusoidal exciting voltage and t and t^* are real and retarded times respectively.

Numerical results for a half-wave dipole designed for f_0 show that the duration of the radiated electric field is increased by $0.5/f_0$, when the duration of the input voltage pulse is an integer multiple of T_0 , where $T_0 = 1/f_0$ is the width of a single cycle. The spectral bandwidth of the radiated electric field appears to be narrower than that of the applied voltage. The reason for this phenomenon may be due to the filtering effect of the half-wave dipole.

Although the results obtained here are based on the above mentioned approximation, it is hoped that the present results will be useful in interpreting more accurate results which one could compute by numerical method(s).

II. Analysis

Let us consider a center-fed slender cylindrical antenna of total length $2h$ (Fig. 1) in free space, excited by a sinusoidal pulse generator. The coordinate system has its origin at the center of the antenna. Initially we assume that the generator has zero internal impedance and the antenna is a perfect conductor. It can be shown (5, 6) that in the Fourier-transform domain (frequency domain) the current distribution $\hat{I}(z, \omega)$ along the antenna (oriented in the z -direction) obeys the following integral equation

$$(i\mu_0 c/4\pi) \int_{-h}^h \hat{I}(z', \omega) \{ \exp(-i\beta R)/R \} dz' = c_1(\omega) \cos \beta z + \frac{\hat{V}(\omega)}{2} \sin(\beta|z|), \quad (1)$$

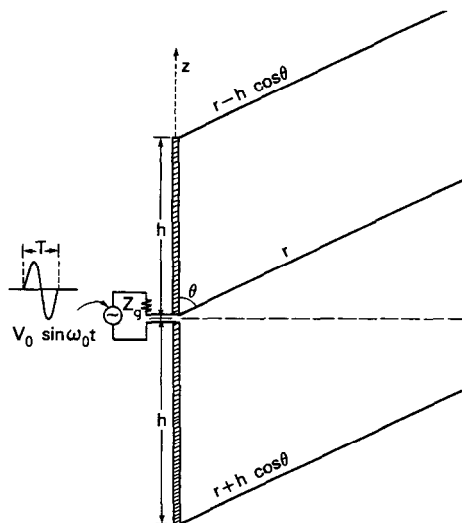


FIG. 1. Transient radiation from a vertical dipole antenna oriented along the z -axis.

where

$$R = [(z - z')^2 + a^2]^{1/2}, \quad (2a)$$

$$a = \text{radius of the thin cylinder}, \quad (2b)$$

$$c = \text{velocity of light}, \quad (2c)$$

$$\beta = \omega/c. \quad (2d)$$

Both z and z' lie along the axis of the antenna, where the current is assumed to be concentrated. The constant $c_1(\omega)$ is to be determined from the boundary conditions at the ends of the antenna. $\hat{V}(\omega)$, the Fourier transform of the sinusoidal input voltage pulse, is given by

$$\begin{aligned} \hat{V}(\omega) &= V_0 \int_{-\infty}^{\infty} \sin \omega_0 t [U(t) - U(t - T)] e^{-i\omega t} dt \\ &= V_0 \int_0^T \sin \omega_0 t e^{-i\omega t} dt \\ &= -[V_0 \omega_0 / (\omega^2 - \omega_0^2)] \cdot [1 - F(\omega, T) e^{-i\omega T}], \end{aligned} \quad (3a)$$

where

$$F(\omega, T) = \cos \omega_0 T + i(\omega/\omega_0) \sin \omega_0 T, \quad (3b)$$

$$T = \text{duration of the pulse}, \quad (3c)$$

$$\omega_0 = \text{carrier frequency (angular) of the pulse}, \quad (3d)$$

$$V_0 = \text{amplitude of the voltage which represents a slice generator at } z = 0, \quad (3e)$$

and where $U(t)$ and $U(t-T)$ are unit step functions. Several simplifying assumptions as mentioned earlier are made (5, 6) in obtaining the integral Eq. (1), but yet no closed form solution is available. Equation (1) then can be solved either analytically by an iteration method (5, 6) which produces an approximate analytical expression, or numerically (5). There is an advantage in obtaining an analytical expression for the current $\hat{I}(z, \omega)$, since the corresponding result could be interpreted more easily. The analytical expression obtained by the method of iteration contains terms with integer powers of $(1/\Omega)$, where $\Omega = 2 \ln(2h/a)$ is much larger than unity. The degree of approximation depends on retaining the highest power of $(1/\Omega)$. The lowest order approximation containing only the first power of $(1/\Omega)$ is known as the zero-order approximation (6). The zero-order solution of (1) has the simple form (5, 6),

$$\hat{I}_0(z, \omega) = [i2\pi/(\Omega\zeta_0)] \cdot \hat{V}(\omega) \cdot [\sin \beta(h-|z|)/\cos \beta h], \quad (4)$$

where $\zeta_0 = \mu_0 c$ = free space impedance. In the first-order approximation, the factor $\sin \beta(h-|z|)/\cos \beta h$ is replaced by $[\sin \beta(h-|z|) + b_1(z, \omega)/\Omega]/[\cos \beta h + d_1(h, \omega)/\Omega]$. Explicit expressions for $b_1(z, \omega)$ and $d_1(h, \omega)$ can be found in Refs (5, 6). Since a pulse contains a broad frequency spectrum, one should take higher-order approximations of the current $\hat{I}(z, \omega)$, so that it represents accurately the desired frequency spectra. However, the higher-order approximations introduce computational complexity. For example, if one wishes to consider the first-order approximation for the current in the frequency domain, then the zeros of the factor $\cos \beta h + d_1(h, \omega)/\Omega$ will introduce complex poles in the process of performing inverse Fourier transform of (4). Thus the zeros of $\cos \beta h + d_1(h, \omega)/\Omega = 0$ may be called complex resonance frequencies, which exhibit a radiation damping effect. However, there is no closed-form solution to this equation for the complex resonance frequency. One may, though, state theoretically that ω_n ($n = 1, 2, \dots$), complex numbers, are solutions of $\cos \beta h + d_1(h, \omega)/\Omega = 0$, and then derive a formal residue expression contributed by these complex poles or complex resonance frequencies ω_n . Since the computation of ω_n must be done numerically, the corresponding inverse transform will require extensive numerical computations. This is one of the main reasons why we shall consider here only the zero-order approximation given by (4), the use of which permits the analytical computations of the corresponding time-domain current and radiated field exactly; and consequently simple interpretations of the results can be offered. It is, however, expected that even if one wishes to compute the time-domain current and transient radiated field more accurately by numerical means (such as the moment method), the present result obtained by using (4) could be used as a helpful guide for the purpose of comparison, since the present result will exhibit the most salient characteristics of the transient radiation of a sinusoidal pulse from a dipole antenna. It may be noted that radiation damping may not be important for radiation from a thin perfectly conducting dipole antenna, when the observation point is at a great distance from the antenna. In this situation "early-time" (which is on the order of h/c) response is not significant.

Although we have discussed the difficulty in retaining the first-order approximation of the current in the frequency domain, the consideration of the zero-order

approximation given by (4) is not trouble free either. We now, therefore, proceed to discuss how to remove the difficulty associated with the zero-order approximation of current in a meaningful way. This expression resembles the current distribution along an open-circuited transmission line of length h and characteristic impedance $Z_0 = \Omega \zeta_0 / 2\pi$, excited by a voltage generator (with zero internal impedance) delivering a voltage which has a Fourier transform $\hat{V}(\omega)$ (5). Since the antenna is near resonance at the frequencies given by $\cos \beta h = 0$ [see Eq. (4)], and if this resonant frequency coincides with the carrier frequency ω_0 , then the expressions (3a) and (4) show that the zero-order current $\hat{I}_0(z, \omega)$ will introduce two second-order poles at $\omega = \pm \omega_0$ during the process of contour integration associated with the inverse Fourier transform of (4) in the complex ω -plane. Although $\hat{V}(\omega)$ is finite at $\omega = \pm \omega_0$, it is necessary to split $\hat{V}(\omega)$ into two parts, which have different analytic properties at infinity, and thereby creating poles during the process of contour integration. Physically this behavior of $\hat{I}_0(z, \omega)$ implies that both the time-domain current and the radiated field increase with time, like $t \sin \omega_0 t$, which cannot happen in practice. This non-physical result is contributed primarily by the unrealistic assumption of a voltage generator having a zero internal impedance, since we have also assumed that the antenna is made of a perfect conductor. This situation is analogous to the transient response of a circuit consisting of a lossless inductance L and a capacitance C connected in series with a voltage generator (with zero internal impedance) producing a sinusoidal pulse $\sin \omega_0 t$ of duration T . If the circuit parameters L and C are such that $\omega_0^2 = (1/LC)$, then the transient current will behave like $t \sin \omega_0 t$. Since, in reality, there is no inductance without resistance and no generator with zero internal impedance, such behavior of the current never occurs. Therefore, in order to be consistent with the zero-order approximation of the current distribution (4) and the assumption of a perfectly conducting thin cylindrical antenna, an inclusion of a non-zero internal impedance of the pulse generator in the present analysis is a meaningful and realistic way to remove the non-physical transient response. Although the internal impedance of a generator is a function of frequency, in general, we shall assume it to be independent of frequency. Let us, therefore, take the generator impedance, $Z_g = \alpha Z_0$, where α is real, ($0 < \alpha < 1$) and Z_0 is defined above. As a result the current distribution $\hat{I}_0(z, \omega)$ is modified as follows:

$$\hat{I}_{0,M}(z, \omega) = [\hat{V}(\omega)/Z_0] \cdot [\sin \beta(h - |z|)] / \{\alpha \sin \beta h - i \cos \beta h\}, \quad (5a)$$

$$= [2i\hat{V}(\omega)/Z_0(1 + \alpha)] \cdot [\sin \beta(h - |z|)] / \{1 + \Gamma \exp(-i2\beta h)\} \cdot \exp(-i\beta h), \quad (5b)$$

$$= [2i\hat{V}(\omega)/Z_0(1 + \alpha)] \cdot [\sin \beta(h - |z|) \cdot \exp(-i\beta h)] \cdot \sum_{n=0}^{\infty} (-\Gamma)^n \exp(-i2n\beta h), \quad (5c)$$

where $\Gamma = (1 - \alpha)/(1 + \alpha) < 1$, is the reflection coefficient from the antenna to the generator. For a matched system $\alpha = 1$ and Γ becomes zero. The relation in (5) for the modified zero-order frequency response of the current was also used in Ref.

(1). This modified expression for the current is adequate when the “early-time” response is not important and one is interested in the far field only.

The radiated electric field in the frequency, or Fourier-transform, domain due to a center-fed thin cylindrical antenna with current distribution given by (5), can be expressed in the following way :

$$\hat{E}_\theta(r, \theta, \omega) = [i\mu_0\omega \sin \theta/4\pi r] \cdot \exp(-i\beta r) g_{0,M}(\theta, \omega), \quad (6)$$

where

$$g_{0,M}(\theta, \omega) = \int_{-h}^h \hat{I}_{0,M}(z', \omega) \exp(i\beta z' \cos \theta) dz' \quad (7a)$$

$$= (4i\hat{V}(\omega)/[Z_0(1+\alpha)]) \cdot [\{\cos(\beta h \cos \theta) - \cos \beta h\}/(\beta \sin^2 \theta)] \\ \cdot \sum_{n=0}^{\infty} (-\Gamma)^n \exp\{-i(2n+1)\beta h\}. \quad (7b)$$

Then the time-dependent radiated electric field given by the inverse Fourier transform of (6), can be expressed formally by

$$E_\theta(r, \theta, t) = (1/2\pi) \cdot \int_{-\infty}^{\infty} \hat{E}_\theta(r, \theta, \omega) \exp(i\omega t) d\omega. \quad (8)$$

The contour C runs along the real axis of the complex ω -plane with indentation from below the poles $\omega = \pm\omega_0$. Finally the contour is closed by a semi-circle in the upper-halfplane at infinity, so that the poles $\omega = \pm\omega_0$ lie inside this closed contour. Then the time-dependent radiated electric field can be expressed as follows :

$$E_\theta(r, \theta, t) = [V_0\zeta_0/\{2Z_0(1+\alpha)\pi r \sin \theta\}] \\ \cdot \left[\sum_{n=0}^{\infty} (-\Gamma)^n \sin \omega_0(t^* - 2nh/c) \cdot [U(t^* - 2nh/c) - U(t^* - 2nh/c - T)] \right. \\ - \sum_{n=0}^{\infty} (-\Gamma)^n \sin \omega_0\{t^* - (2nh + h(1 - \cos \theta))/c\} \\ \cdot [U\{t^* - (2nh + h(1 - \cos \theta))/c\} - U\{t^* - (2nh + h(1 - \cos \theta))/c - T\}] \\ - \sum_{n=0}^{\infty} (-\Gamma)^n \sin \omega_0\{t^* - (2nh + h(1 + \cos \theta))/c\} \\ \cdot [U\{t^* - (2nh + h(1 + \cos \theta))/c\} - U\{t^* - (2nh + h(1 + \cos \theta))/c - T\}] \\ + \sum_{n=0}^{\infty} (-\Gamma)^n \sin \omega_0\{t^* - 2(n+1)h/c\} \cdot [U\{t^* - 2(n+1)h/c\} \\ \left. - U\{t^* - 2(n+1)h/c - T\}] \right], \quad (9)$$

where $t^* = t - r/c =$ retarded time. Similarly, the time-dependent current dis-

tribution along the dipole corresponding to the frequency domain zero-order modified expression given by (5) can be represented as follows:

$$I_{0,M}(z, t) = (1/(2\pi i)) \cdot \int_C \hat{I}_{0,M}(z, -is) e^{st} ds, \quad (10)$$

$$\begin{aligned} &= [V_0/Z_0(1+\alpha)] \cdot \left[\sum_{n=0}^{\infty} (-\Gamma)^n \sin \omega_0 \{t + (h-|z| - (2n+1)h)/c\} \right. \\ &\quad \cdot [U\{t + (h-|z| - (2n+1)h)/c\} - U\{t + (h-|z| - (2n+1)h)/c - T\}] \\ &\quad - \sum_{n=0}^{\infty} (-\Gamma)^n \sin \omega_0 \{t - (h-|z| + (2n+1)h)/c\} \\ &\quad \cdot [U\{t - (h-|z| + (2n+1)h)/c\} - U\{t - (h-|z| + (2n+1)h)/c - T\}] \left. \right]. \end{aligned} \quad (11)$$

Although the exciting sinusoidal voltage has a finite duration T , both the time-dependent radiated electric field and the current along the dipole continue to exist for a longer period with diminished amplitudes due to multiple reflections (ringing) between the dipole antenna and the pulse generator. However, when the pulse generator is matched to the antenna, then $\alpha = 1$ and $\Gamma = 0$. Therefore, the corresponding time-dependent radiated field and the antenna current distribution expressions simplify themselves considerably as shown below, i.e.

$$\begin{aligned} E_\theta(r, \theta, t) &= [V_0 \zeta_0 / (4Z_0 \pi r \sin \theta)] \cdot [\sin \omega_0 t^* \cdot \{U(t^*) - U(t^* - T)\} \\ &\quad - \sin \omega_0 \{t^* - h(1 - \cos \theta)/c\} \cdot [U\{t^* - h(1 - \cos \theta)/c\} - U\{t^* - h(1 - \cos \theta)/c - T\}] \\ &\quad - \sin \omega_0 \{t^* - h(1 + \cos \theta)/c\} \cdot [U\{t^* - h(1 + \cos \theta)/c\} - U\{t^* - h(1 + \cos \theta)/c - T\}] \\ &\quad + \sin \omega_0 (t^* - 2h/c) \cdot \{U(t^* - 2h/c) - U(t^* - 2h/c - T)\}] \end{aligned} \quad (12)$$

$$\begin{aligned} I_{0,M}(z, t) &= (V_0/2Z_0) \cdot [\sin \omega_0 (t - |z|/c) \cdot \{U(t - |z|/c) - U(t - |z|/c - T)\} \\ &\quad - \sin \omega_0 \{t - (2h - |z|)/c\} \cdot \{U(t - (2h - |z|)/c) - U(t - (2h - |z|)/c - T)\}]. \end{aligned} \quad (13)$$

The first term of the radiated field (12) resembles very much the sinusoidal voltage pulse, but retarded, and is radiated directly from the feed-point $z = 0$ of the dipole antenna. The second term of (12) represents radiation from the upper end $z = h$ of the dipole, whereas the third term can be identified as the radiated field from the lower end $z = -h$. The fourth or last term of (12) is again radiated from the feed-point $z = 0$, after the current undergoes reflection from $z = \pm h$. In other words, one half of the last term of (12) is contributed by the current which traveled from $z = 0$ to $z = h$ and then back to $z = 0$, and the other half is due to the current traveling from $z = 0$ to $z = -h$ and then back to $z = 0$. Thus, there are five overlapping radiated pulses, of which two look alike, because of their same phase retardation. This behavior of the field indicates that radiation appears to emerge only from the discontinuities of the antenna.

The first term of (13) represents a current pulse at a given point $\pm z$ of the dipole

antenna, for which the pulse has not yet reached the end points of the antenna. It may be noted here that time t in the expression for the current (13) is the real time or the retarded time at $r = 0$. The second term in (13) designates the current pulse at a point $\pm z$ on the antenna, after it has been reflected once from the end points $z = \pm h$. The minus sign with the second term shows that the reflection coefficient of the end point is -1 .

Assuming that $T > 2h/c$, the relation (12) can be re-expressed using seven intervals, in some of which two or more pulses overlap. Similarly, the current can be represented by using three intervals. Such representations which are convenient for numerical computations can be found elsewhere (7). It is interesting to note that instead of taking $V(t) = V_0 \sin \omega_0 t \cdot [U(t) - U(t - T)]$ as the exciting input voltage pulse, taking $V(t) = \mathcal{V}(t) \cdot [U(t) - U(t - T)]$, where $\mathcal{V}(t)$ is an integrable function of t , then the expressions for the time dependent radiated electric field and the antenna current [see Eqs (8) and (10)], respectively, could have also been represented in a general fashion in the following manner. For the sake of simplicity we shall present the results for $\alpha = 1$ and $\Gamma = 0$. In order to facilitate the integration process, we shall use the following two relations which can be easily proved, i.e.

$$L^{-1}[\exp(-st_0) \cdot L\{g(t)\}] = g(t - t_0) \cdot U(t - t_0), \quad (14a)$$

$$L^{-1}[\exp\{-s(t_0 + T)\} \cdot L\{g(t + T)\}] = g(t - t_0) \cdot U(t - t_0 - T), \quad (14b)$$

where the symbols L and L^{-1} stand for the Laplace transform operation and its inverse respectively. Then we have

$$\begin{aligned} E_\theta(r, \theta, t)/[\zeta_0/(4Z_0\pi r \sin \theta)] &= \mathcal{V}(t^*) \cdot [U(t^*) - U(t^* - T)] \\ &- \mathcal{V}\{t^* - h(1 - \cos \theta)/c\} \cdot [U\{t^* - h(1 - \cos \theta)/c\} - U\{t^* - h(1 - \cos \theta)/c - T\}] \\ &- \mathcal{V}\{t^* - h(1 + \cos \theta)/c\} \cdot [U\{t^* - h(1 + \cos \theta)/c\} - U\{t^* - h(1 + \cos \theta)/c - T\}] \\ &+ \mathcal{V}(t^* - 2h/c) \cdot [U(t^* - 2h/c) - U(t^* - 2h/c - T)], \end{aligned} \quad (15)$$

and

$$\begin{aligned} I_{0,M}(z, t)/(2Z_0) &= \mathcal{V}(t - |z|/c) \cdot [U(t - |z|/c) - U(t - |z|/c - T)] \\ &- \mathcal{V}\{t - (2h - |z|)/c\} \cdot [U\{t - (2h - |z|)/c\} - U\{t - (2h - |z|)/c - T\}]. \end{aligned} \quad (16)$$

Observing now the similarities between (12) and (15), and (13) and (16) respectively, the results of (15) and (16) can be interpreted in the same manner.

III. Some Numerical Results for a Half-wave Dipole

When the input voltage is a sinusoidal pulse of a single cycle, then the pulse length T_0 is given by $\omega_0 T_0 = 2\pi$, where $\omega_0 = 2\pi f_0$ is the sinusoidal carrier frequency (circular). For a half-wave center-fed dipole of total length $2h$ designed for the

carrier frequency f_0 one finds $\omega_0 h/c = \pi/2$ or $h/c = T_0/4$. Figure 2 shows the normalized time dependent modified current $I_{0,M}(z, t)/[V_0/2Z_0]$ along the half-wave dipole as a function of a normalized time tf_0 , where t is the real time. The horizontal line passing through 0 shows that no current flows at the antenna end points $|z| = h$. The duration of the total current pulse observed at $|z| = 0$ is $1.5T_0$, whereas the exciting voltage pulse width is T_0 . However, as the observation point $|z|$ along the dipole increases, the duration of the current pulses decreases, approaching zero as $|z| \rightarrow h$. The current pulse at any point $|z|$ is even symmetric about the normalized time $tf_0 = 1.5/2$, when the current attains a negative maximum for all $|z| \neq h$. The negative maximum appears to be higher in magnitude than the two positive maxima for all $|z| \neq h$. In particular, the two positive maxima at $z = 0$ occur at $tf_0 = 0.25$ and 1.25 respectively.

In Fig. 3 the normalized time dependent radiated electric field, $E_\theta(r, \theta, t)/[V_0 \zeta_0/(4\pi Z_0 r)]$ is shown as a function of normalized retarded time t^*f_0 for four different observation angles $\theta = 30^\circ, 45^\circ, 60^\circ$ and 90° , which are measured from the axis (see Fig. 1) of the dipole. As expected the broadside ($\theta = 90^\circ$) field is the strongest one. For all angles θ the electric field shows an odd symmetry about $t^*f_0 = 1.5/2$. For each angle of observation the magnitude of the normalized electric field is maximum around the time $f_0 t^* = 0.5$ and 1 . Although the exciting sinusoidal voltage pulse has a length T_0 (for a single cycle), the composite radiated time-dependent electric field (like the current pulse along the dipole at a given point) has a time duration of $1.5T_0$, i.e. the radiated signal is stretched 1.5 times compared to the exciting voltage pulse. This phenomenon of broadening of the radiated field is caused by the reflection of current pulses from the end points of the dipole. It may be noticed that if the duration of the input voltage pulse is nT_0 , an

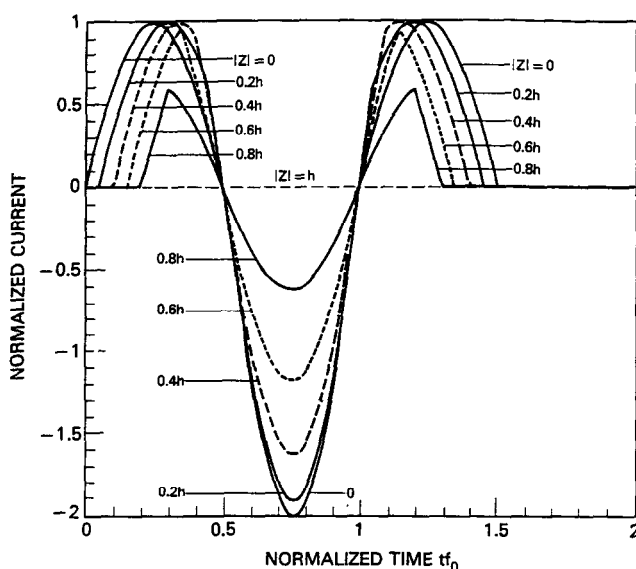


FIG. 2. Normalized transient current along a half-wave dipole.

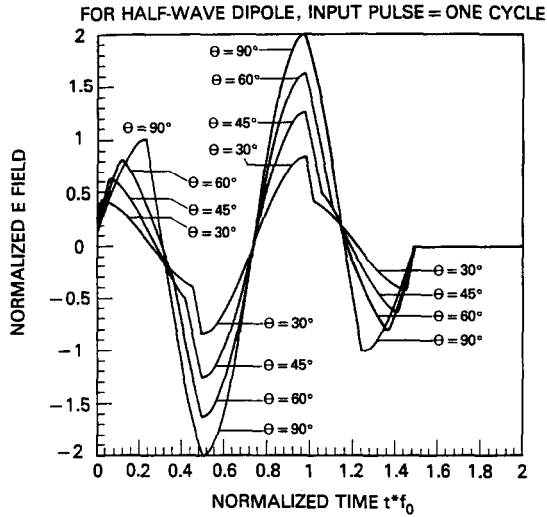


FIG. 3. Transient radiated electric field observed at different angles from a half-wave dipole.

integer multiple of T_0 , then (see Fig. 4) the duration of the corresponding radiated field becomes $(n + 1/2)T_0$ only. This increased duration appears as distortions in the beginning and end of the respective radiated pulses. Figure 5 shows the frequency response of the radiated electric field at broadside observation as a function of the

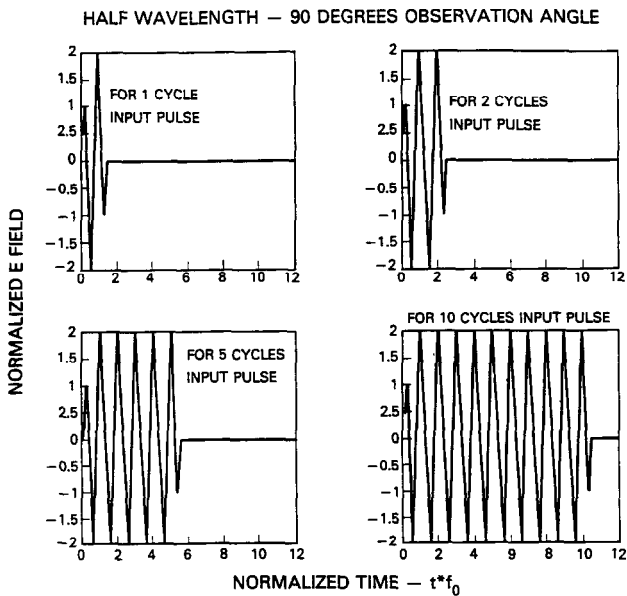


FIG. 4. Transient radiated electric field for input pulses with different widths. Half-wave dipole. Observation angle = 90° .

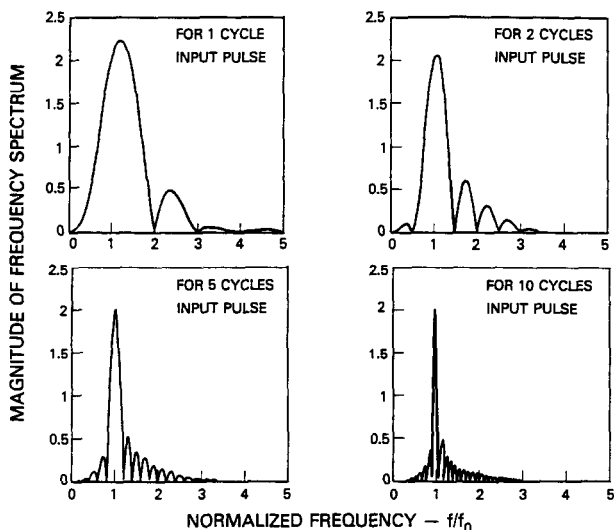


FIG. 5. Frequency response of radiated electric field for input pulses with different widths. Half-wave dipole. Observation angle = 90° .

normalized frequency f/f_0 for four different input pulses of durations T_0 , $2T_0$, $3T_0$ and $4T_0$. The maximum amplitudes of these radiated fields occur at $f > f_0$, but they approach $f/f_0 = 1$ as the duration of the input voltage pulses is increased. This phenomenon can also be observed from Figs 6 and 7, where the frequency responses of the input applied voltage and the radiated electric field are compared. Figures 6 and 7 drawn for $2h/a = 75$, indicate [see Eq. (6)] that the normalized radiated electric field $|r\hat{E}_\theta(r, \theta, \omega)|$, which attenuates more for higher values of $\Omega = 2 \ln(2h/a)$ has a frequency bandwidth relatively narrower than that of the

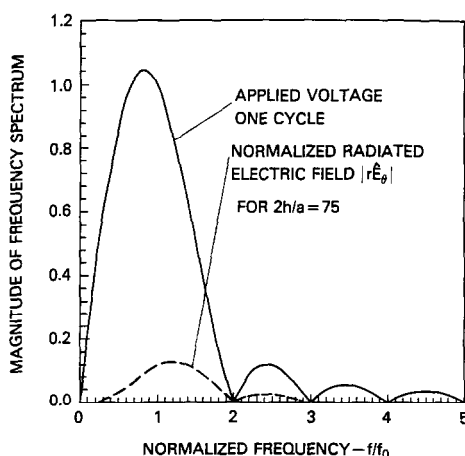


FIG. 6. Spectra of applied voltage (1 cycle) and radiated electric field for a half-wave dipole. Observation angle = 90° .

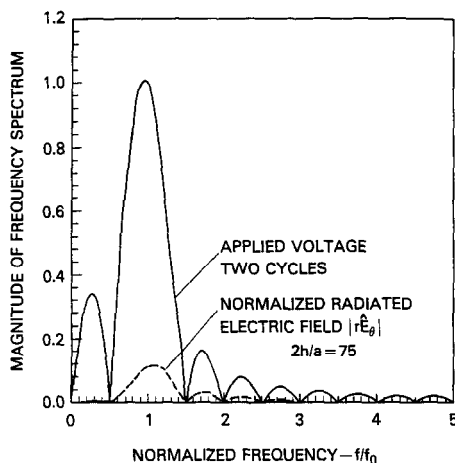


FIG. 7. Spectra of applied voltage (2 cycles) and radiated electric field for a half-wave dipole. Observation angle = 90° .

applied voltage. The reason is that the antenna is assumed to be very thin, which in general has a narrow bandwidth. It may also be noticed that the antenna is attenuating more at lower frequencies than at higher ones.

IV. Conclusion

A modified zero-order approximate solution is used for the frequency or Fourier-transform domain current distribution along a dipole antenna. It is shown that when the dipole is matched to and is excited by a sinusoidal pulse generator, delivering a voltage $\sin \omega_0 t$ of duration T , the corresponding time dependent antenna current as well as the radiated electric field are broadened in time longer than T . This phenomenon is caused by the reflection of time dependent current from the end points of the antenna. In particular, when the duration of the input voltage is an integer multiple of T_0 , where T_0 is the period of a single cycle, the duration of the radiated electric field is increased by $0.5T$. At a given point $|z|$ on any half-section of the dipole the total current consists of two pulses, one is incident from $|z| = 0$ and the other arrives at $|z|$ after undergoing a reflection from the end-point. The duration of the current pulse along the antenna decreases as the observation point on the antenna increases. The current pulse at a given point $|z|$ of the antenna possesses an even symmetry about the dimensionless time $tf_0 = 1.5/2$. The corresponding radiated electric field consists of five signals, each of duration $1/f_0$, which overlap in time. Since two of these signals have the same phase, only four signals are apparent. Of these four signals, the first one resembles very much the input sinusoidal voltage, but retarded, and is radiated directly from the feed point $z = 0$ of the dipole antenna. The second and the third signals radiate from the end points $z = h$ and $z = -h$ respectively of the dipole, and, therefore, delay in time with respect to the first one by $h(1 - \cos \theta)/c$ and $h(1 + \cos \theta)/c$. The

direction of the observation, given by θ , is measured from the axis of the half-wave dipole. The fourth signal delayed in time by $2h/c$ with respect to the first one, is also radiated from the feed-point $z = 0$, after the current undergoes reflection from two ends $z = \pm h$. In other words, one half of the fourth signal is contributed by the current which traveled from $z = 0$ to $z = h$ and then back to $z = 0$, and the other half is due to the current traveling from $z = 0$ to $z = -h$ and then back to $z = 0$ [see Eq. (12)]. This observation indicates that radiation originates from the discontinuous points of the antenna. It may be noted also that the total radiated electric field for any angle of observation has an odd symmetry about the normalized retarded time $t^*f_0 = 1.5/2$. The spectra of the applied voltage signal and the corresponding radiated electric field do not have their maxima at the same frequency. The peak value of the frequency spectra of the radiated electric field occurs at a frequency $f > f_0$. If the duration of the applied signal is increased, the peak value of the radiated spectra approaches the peak of the applied voltage spectrum at the carrier frequency $f = f_0$. The bandwidth of the radiated normalized electric field is narrower than that of the applied voltage. The reason may be that the antenna is assumed to be thin, which in general has a narrow bandwidth. The antenna attenuates more at lower frequencies than at higher ones.

Since the results presented here are based on an approximate expression of the current, some of the observations noted above may not be precise. Nevertheless, it is believed that they will be helpful in interpreting more accurate results which may be obtained by numerical methods.

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